Engineering Notes

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Turbulence for Flight Simulation

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Nomenclature

$C_{ij}(x,r)$ = second-order, two-point velocity correlation tensor for nonhomogeneous turbulence
$E(t)$ = symbol defined in Eq. (6)
$f \sim f(r,t)$ = longitudinal correlation function for isotropic turbulence
$k \sim k(r,t)$ = single function required to specify third-order velocity product moments
$K(t)$ = symbol defined in Eq. (3)
$r = |r|$ = magnitude of separation vector
$S_{ilj}(x,r)$ = third-order, two-point velocity product correlation tensor for nonhomogenous turbulence
u_i = turbulent velocity
U_i = flow velocity (total)
U = forward flight speed of aircraft
$\alpha(t)$ = symbol defined in Eq. (6)
$\sigma \sim \sigma(t)$ = intensity of turbulence
$\lambda \sim \lambda(t)$ = dissipation scale of turbulence
$\Lambda \sim \Lambda(t)$ = integral scale of turbulence
ν = kinematic viscosity
$\chi(t)$ = symbol defined in Eq. (6)

Introduction

FAITHFUL simulation of atmospheric turbulence is a problem of great concern to the aviation industry in general. In the early years of this discipline, commonly used methods for artificially generating turbulence time-histories employed a Gaussian model of turbulence, but in recent years the trend has been toward more realism with a modified Gaussian model, where fourth-order moments are not obtained directly from the second-order moments (see Ref. 1 and the references cited therein). However, none of these two fundamental models produce odd-order moments (skewness, superskewness, etc.[2]) in the simulated "turbulence" and the net result is a random time history that pilots who "fly" through this simulated disturbance, in state-of-the-art flight simulators, often describe as too "continuous" and too "monotonous" when compared to real atmospheric turbulence. Pilots refer to these collective features in the simulated time histories as "lacking the (all-important) element of surprise" of authentic hydrodynamic turbulence. In view of this discrepancy between (as currently) simulated and real turbulence, this Note proposes to establish the explicit role of third-order moments in the dynamical characteristics of the correlation structure of atmospheric turbulence. Hopefully, this result will stimulate the search for non-Gaussian turbulence models that duly reflect the true nature of the phenomenon. Briefly, the role of third-order moments is to provide a "driving force" for the turbulence two-point velocity correlations, a force that inherently produces its characteristically unique feature(s).

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The Role of Third-Order Moments

The dynamical equation of turbulence is the well-known Kármán-Howarth Equation,[3]

$$\frac{\partial(\sigma^2 f)}{\partial t} - \sigma^3\left(\frac{\partial}{\partial r}+\frac{4}{r}\right)k = 2\nu\sigma^2\left(\frac{\partial^2}{\partial r^2}+\frac{4}{r}\frac{\partial}{\partial r}\right)f \tag{1}$$

$\sigma \sim \sigma(t)$ denotes the intensity of the turbulence, $f \sim f(r,t)$ the longitudinal correlation function, and $k \sim k(r,t)$ the single function required to completely specify third-order velocity moments such as $\langle u_i(x,t)u_j(x,t)u_l(x+r,t)\rangle$; ν, the kinematic viscosity of the fluid. Note that third-order moments are indeed important in the turbulence structure, as evidenced by the presence in Eq. (1) of the nonzero term, $\sigma^3[(\partial/\partial r)+(4/r)]k$. A time-dependent equation is obtained from Eq. (1) by integrating it in r from $r=0$ to ∞; accordingly,

$$\int_0^\infty \frac{\partial}{\partial t}(\sigma^2 f)\mathrm{d}r = \frac{\mathrm{d}}{\mathrm{d}t}(\sigma^2\Lambda) \tag{2}$$

where $\Lambda = \int_0^\infty f \mathrm{d}r$,

$$\sigma^3\int_0^\infty\left(\frac{\partial}{\partial r}+\frac{4}{r}\right)k\mathrm{d}r = \sigma^3 K(t) \tag{3}$$

and

$$2\nu\sigma^2\int_0^\infty\left(\frac{\partial^2}{\partial r^2}+\frac{4}{r}\frac{\partial}{\partial r}\right)f\mathrm{d}r = 8\nu\sigma^2\int_0^\infty r^{-1}\frac{\partial f}{\partial r}\mathrm{d}r \tag{4}$$

An analytical estimate to this last expression is provided by writing $f(r,t) \simeq \exp(-r^2/2\lambda^2)$, where $\lambda \sim \lambda(t)$ is the dissipation scale (Ref. 3, p. 47, 94) of the turbulence. For analytical purposes, this approximation to $f(r,t)$ is more appropriate than is the standard $f(r,t) \simeq \exp(-r/\Lambda)$ and it readily follows that

$$\int_0^\infty r^{-1}\frac{\partial f}{\partial r}\mathrm{d}r \simeq -\frac{\Lambda}{\lambda^2}$$

The intended purely time-dependent form of Eq. (1) is therefore

$$\frac{\mathrm{d}(\sigma^2\Lambda)}{\mathrm{d}t} + \left(\frac{8\nu}{\lambda^2}\right)\sigma^2\Lambda = \sigma^3 K(t) \tag{5}$$

Setting $\chi(t)=\sigma^2\Lambda$ and $\alpha(t)=(8\nu/\lambda^2)$ converts Eq. (5) into a kind of Langevin equation,

$$\frac{\mathrm{d}}{\mathrm{d}t}\chi(t)+\alpha\chi(t)=E(t) \tag{6}$$

where $E(t)=\sigma^3 K(t)$ denotes an external excitation ("Langevin force"). This excitation comes from third-order velocity moments only and is not a "white-noise" type of forcing function. This explicit and rather marked dependence of the correlation structure on the third-order moments (third-order moments are sometimes called "turbulence self-interaction") is what separates turbulence from other naturally occurring random phenomena and also what conveys upon it, its most interesting feature. Indeed, without this dependence, the study of turbulence reduces to that of classical (Gaussian) diffusion and it has already been established (see the introduction) that such analyses do not produce satisfactory, or even acceptable, results.

The solution to Eq. (6) is[4]

$$\chi(t) = e^{-\int\alpha(t)\mathrm{d}(t)}\left[\int e^{\int\alpha(t)\mathrm{d}t}E(t)\mathrm{d}t + C\right] \tag{7}$$

where C is a constant determined from initial conditions. Equation (7) establishes the role of third-order velocity moments in the time dependence of $\chi(t) = \sigma^2(t)\chi(t)$ and also their intimate relationship with the "viscous-friction factor," $\alpha(t) = 8\nu/\lambda^2$. Without them, i.e., when $E(t) \simeq 0$, the solution to Eq. (6) is simply

$$\chi(t) = Ce^{-\int\alpha(t)\mathrm{d}t} \tag{8}$$

a result with no obviously interesting characteristics. In contradistinction, Eq. (7) exhibits a feature bearing further investigation. To be sure, if $\alpha(t)$ is negligibly small (viz., "high Reynold number" turbulence), $\chi(t)$ then reduces to $\chi(t) \simeq \int E(t)\mathrm{d}t + C$; If $E(t)$ is also some constant,[5] say E_0, the solution becomes

$$\chi(t) \simeq E_0 t + C \tag{9}$$

Equation (9) suggests that, under appropriate conditions, rapid increases in turbulence intensity, e.g., $\mathrm{d}[\sigma^2(t)]/\mathrm{d}t \sim t$, are automatically accompanied by decreases in the integral scale. The overall result of this somewhat typical combination is a turbulence that is highly random and has high intensity ("severe" turbulence), while the completely reverse process produces a relatively "moderate" turbulence. Through this type of analysis, the potential role of third-order velocity moments in augmenting, or perhaps even themselves producing, the "patchy" structure of turbulence, where periods of intense turbulence are followed by relatively quiet periods, is undeniably established.

Application to an Airplane Flying through "Frozen" Nonhomogeneous Turbulence

For an airplane flying along a horizontal path (i.e., in the x direction through "frozen" *nonhomogeneous turbulence*, the turbulence equations of motion with respect to a fixed (ground-based) set of axes are

$$\frac{\partial(U_i U_l)}{\partial x_l} = \nu\frac{\partial^2 U_i}{\partial x_l \partial x_l} \tag{10}$$

where the pressure gradient terms have been neglected. Taking the mean flow velocity to be zero, or at the very least "small" in comparison to the forward flight speed U of the aircraft, the Navier-Stokes equation with respect to the aircraft set of axes reduces to

$$-U\frac{\partial u_i}{\partial x} + \frac{\partial(u_i u_l)}{\partial x_l} = \nu\frac{\partial^2 u_i}{\partial x_l \partial x_l} \tag{11}$$

Here the "convection" term $(-U\partial u_i/\partial x)$ rather naturally converts into $(\partial u_i/\partial t)$, i.e., $[-U(\partial u_i/\partial x)] \sim (\partial u_i/\partial t)$, and ultimately all correlations in the two-point stochastic-averaged equation convert analogously, viz,

$$C_{ij}(x,r) = \langle u_i(x)u_j(x+r)\rangle \sim C_{ij}(t,r) \tag{12}$$

$$S_{ilj}(x,r) = \langle u_i(x)u_l(x)u_j(x+r)\rangle \sim S_{ilj}(t,r) \tag{13}$$

etc. In this way, the "frozen" nonhomogeneous turbulence problem, where all correlations are functions of x (and r), is brought into line with the fundamental problem of homogeneous isotropic "nonfrozen" turbulence, where all correlations are functions of time. Equation (11), with $[-U(\partial u_i/\partial x)]$ replaced by $(\partial u_i/\partial t)$, is the equation from which the Kármán-Howarth equation, Eq. (1), is under appropriate constraints eventually derived (Ref. 3, p. 100).

Conclusions

Third-order velocity product moments are indeed important in the correlation structure of turbulence. This result requires that these moments be explicitly included in any faithful simulation of same. These moments ingress as a "driving force" into the differential equation for the two-point velocity correlation of the turbulence and as much convey upon it, its characteristically unique features. Without them, the dynamical aspects of turbulence reduce to those of Gaussian diffusion and unjustly purloin from it its natural "self-interaction," as well as its interaction with the underlying mean flow. Recall that these "interactions," arise from the nonlinear convective term in the Navier-Stokes equation. Under select circumstances, third-order moments also reinforce the "patchy" nature of turbulence.

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Experimental and Theoretical Study of Wings with Blunt Trailing Edges

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Introduction

WINGS in subsonic flow are characterized by a rounded leading edge and a sharp trailing edge. Currently, considerable research work is being done to increase the lift by cir-

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